

# **Applied Mathematics for Malting and Brewing Technologists**

Technological Calculations, Benchmarks and Correlations for Process Optimization

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## Abbreviations and symbols

**Note:** In all calculations the metric system is used. For the conversion of SI units into other measurement systems see Chapter 19.

abv	alcohol by volume	$E_5$	$0.81 \cdot E_2$
A	area, surface	$E_{PW}$	extract of the pitching wort at 20 °C
$A_C$	alcohol content in % m/m or % v/v	$E_R$	evaporation rate
$A_S$	area of a sphere	FA	filter aid
B	barley	FAN	free amino acids
B.a.GM	barley as green malt	$F^\circ$	degree of fermentation
b	width	$F^\circ_{ap}$	degree of fermentation apparent
BFM	bottle filling machine	$F^\circ_{apf}$	degree of fermentation apparent final
BCM	bottle cleaning machine	$F^\circ_{aplc}$	degree of fermentation in the lager cellar
BU	EBC bitterness unit	$F^\circ_{real}$	degree of fermentation real
c, $c_p$	specific heat capacity	FB	finished beer
CCV	cylindroconical tank	FD	finished drink
cps	cycles per seconds	FM	finished malt
$c_Y$	yeast concentration	FW	finished wort
d	day	g	acceleration of gravity = $9.81 \text{ m/s}^2$
d, $\emptyset$	diameter	GM	green malt
DE	diatomaceous earth	h	height
DFS	dosing filter system	h	hour
DM	dry matter, dry mass	h	enthalpy
$DM_B$	barley dry matter	HE	heat exchange
$DM_M$	malt dry matter	hL	hectoliter
$DM_Y$	yeast dry matter	k	heat transfer coefficient
$DM_{YI}$	yeast dry matter increase	K	temperature in Kelvin
DMS	dimethylsulfide	l	length
e	constant $e = 2.71828$	L	liter
E	energy	m	mass
E	element	$\dot{m}$	mass flow rate
E	evaporation	M	malt
$E_C$	extract content	$M_C$	moisture Content
$E_1$	existing apparent residual extract in percent	MEV	malt equivalent value
$E_2$	already fermented apparent extract in percent	$M_{GL}$	grist load
$E_3$	spindled value of the final fermentation sample in percent	mL	milliliter
$E_4$	still available fermentable residual extract in percent ( $E_1 - E_3$ )	NTP	normal temperature and pressure

OG	original gravity	t	time
OG <sub>PW</sub>	original gravity of the pitching wort	t <sub>B</sub>	average boiling time
OG <sub>CKW</sub>	original gravity of the cold knockout wort	TCC	trough chain conveyor
OG <sub>HKW</sub>	original gravity of the hot knockout wort	UMB	un-malted barley
OG <sub>FB</sub>	original gravity of the finished beer	V	volume
OP	overpressure	$\dot{V}$	volume flow
p	pressure	V <sub>CKW</sub>	volume of cold knockout wort
P	power	V <sub>FW</sub>	volume of first wort
Pe	perimeter	V <sub>HKW</sub>	volume of hot knockout wort
PHE	plate heat exchanger	VKFW	volume of kettle full wort
PU	pasteur Units	V <sub>Ma</sub>	volume of mash
PW	pitching wort	V <sub>PW</sub>	volume of pitching wort
Q	heat quantity	V <sub>SG</sub>	volume of spent grain
Q <sub>set</sub>	set filling amount	W	specific main striking volume
$\dot{Q}$	heat flow rate	W	water
q	specific heat quantity	W <sub>C</sub>	Water content
r	radius	WC	water column
r	heat of evaporation	W <sub>D</sub>	delivery work
RE <sub>FB</sub>	real extract of the finished beer at 20 °C	x	humidity
rpm	rotations per minute	y	year
s	seconds	Y	yield
s	speed	Y <sub>ffm</sub>	yield of fine flour malt in the air-dried state
S°	steeping degree	Y <sub>BH</sub>	brewhouse yield
spec.	specific	Y <sub>Bit</sub>	bitterness utilization in percent
SG	spent grain	Y <sub>eff</sub>	brewhouse efficiency
STHE	shell and tube heat exchanger	Y <sub>OBY</sub>	overall brewhouse yield
STHS	short time heating system / flash pasteurizer	% m/m	% mass/mass
SS	sugar sirup	% v/v	% volume/volume
$\alpha$	angle	$\mu$	micro
$\alpha$	heat transfer coefficient	v	kinematic viscosity
$\Delta$	difference	$\pi$	pi = 3.1416
$\Delta\vartheta$	temperature difference	$\rho$	density
$\eta$	dynamic viscosity	$\sigma$	mechanical tension
$\vartheta$	temperature in degrees Celsius	$\sigma$	population standard deviation
$\lambda$	gas solubility	$\varphi$	relative humidity
$\lambda$	thermal conductivity	$\omega$	angular velocity

Statistics			
$\Delta \bar{x}$	confidence interval	$s^2$	variance
$\bar{x}$	mean	P	statistical certainty
$\sigma^2$	population variance	t	test statistic
Q	test value	CV	coefficient of variation
$r^2$	coefficient of determination (= B)	$a_0$	regression constant
s	standard deviation		
Indices e.g.			
A	air	GR	grist
A	actual	HKW	hot knockout wort
a	average	I	increase
ad	air dried	KF	kettle full wort
ap	apparent	L	laboratory
B	barley	L	losses
B	buffer	M	malt
BH	brewhouse	Ma	mash
Bit	bitterness	MF	main fermentation
bb	barrel	P	pyramid
C	cylinder	PM	pilsner malt
C	content	PW	pitching wort
cal	calculate	R	rate
CA	caramel	req	required
CKW	cold knockout wort	real	real
CM	content malt	S	sphere
Co	cone	SD	spherical dome
CS	conical section	SG	spent grain
CU	cuboid	SM	sour malt
eff	effective	SV	strike volume / water
Ex	extract	SpV	sparging volume
eth	ethanol	t	total
f	final	TE	total evaporation
F	frustum	Tc	truncated cone or frustum
F	factor	V	vapor
FB	finished beer	W	water
FW	first wort	Y	yeast
G	grain		

High abv beer	$\geq 16$ °Plato OG
Low abv beer	$< 11$ °Plato OG
Full beer	$\geq 11$ and $< 16$ °Plato OG
Pilsner type beer	$\approx 12$ °Plato OG

## Overview of calculation examples

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## Preface

The most well known texts on applied mathematics for maltsters and brewers [1], [2], [3] are over 50 years old and no longer meet the requirements of the 21st century. A modernized textbook for brewers and maltsters revised by *R. Simon* [4] was released in 1986 and thus is thirty years old. Furthermore in 2003 the Master Brewers Association of Americas released an interesting handbook for basic brewing calculations [5], but it utilizes only Anglo-American measurement units.

The purpose of this reference book is to provide an overview of technological calculations and guidelines from literature, which are supplemented by self-determined correlations and statistically reliable relationships. These are helpful for trainees, practitioners, and students to optimize process management in beer production.

Furthermore, tables and graphs needed for technological calculations are included in a manner to enable rapid solutions without long searches.

The information required for assessing the results including reference values found in literature are presented without much explanation of the technological, biochemical, microbiological, and technical relationships. For understanding the requisite technology studies referenced modern literature see [6], [7], [8], [9], [10], [11], [12], [13], [14], [15].

In addition to the computational approaches, there are sample calculations with solutions to assist students and skilled workers gain a deeper understanding of the subject matter. From these computational approaches and sample calculations, easy operation-specific tasks can be derived.

Especially for small breweries that do not have large analytical study capacity, simpler technological approximate solutions are proposed.

Finding the solution of these tasks require basic knowledge of the handling of a calculator with integrated trigonometric, logarithmic, and simple statistical functions.

The use of the included equations for creating universally usable calculation documents with the help of Excel<sup>®</sup> is strongly recommended.

Power supply for the brewing and malting industry (heat, cold, and electricity), compressed air supply, and CO<sub>2</sub> recovery are not dealt with in this textbook, because of the extent of information already available in literature, for example, [16], [17], [18] and [19].

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